Computing Heat Transfer Coefficients II

Larry Caretto
Mechanical Engineering 375

Heat Transfer

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California State University Northridge

Outline

- Review last external and introduction to internal flows
- Heat transfer coefficients for internal flows
  - Temperature for computing properties
  - Laminar and turbulent flows
  - Pressure drop and heat transfer
- Circular and non-circular geometries
- Free convection

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Review External Flow Basics

- The flow is unconfined
- Moving objects into still air are modeled as still objects with air flowing over them
- There is an approach condition of velocity, U_∞, and temperature, T_∞
- Far from the body the velocity and temperature remain at U_∞ and T_∞
- T_∞ is the (constant) fluid temperature used to compute heat transfer

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Review Flat Plate Equations

- Laminar flow (Re_x, Re_L < 500,000, Pr > .6)

$$C_{f_x} = \frac{\tau_{wall}}{\rho U_\infty^2/2} = 0.664\,\mathrm{Re}_x^{-1/2} \qquad Nu_x = \frac{h_x x}{k} = 0.332\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}$$

$$C_f = \frac{\bar{\tau}_{wall}}{\rho U_\infty^2/2} = 1.33\,\mathrm{Re}_L^{-1/2} \qquad Nu_L = \frac{\bar{h}L}{k} = 0.664\,\mathrm{Re}_L^{1/2}\,\mathrm{Pr}^{1/3}$$

- Turbulent flow (5×10^5 < Re_x, Re_L < 10^7)

$$C_{f_x} = \frac{\tau_{wall}}{\rho U_\infty^2/2} = 0.059\,\mathrm{Re}_x^{-1/5} \qquad Nu_x = \frac{h_x x}{k} = 0.0296\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3}$$

$$C_f = \frac{\bar{\tau}_{wall}}{\rho U_\infty^2/2} = 0.074\,\mathrm{Re}_L^{-1/5} \qquad Nu_L = \frac{\bar{h}L}{k} = 0.037\,\mathrm{Re}_L^{0.8}\,\mathrm{Pr}^{1/3}$$

For turbulent Nu, .6 < Pr < 60

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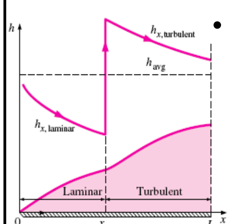
Review Flat Plate Equations II



- Average properties for combined laminar and turbulent regions with transition at x_c = 500000 $\nu/U\infty$
  - Valid for $5\times10^5 < Re_L < 10^7$ and $0.6 < Pr < 60$

$$C_f = \frac{\bar{\tau}_{wall}}{\rho U_\infty^2/2} = \frac{0.074}{\mathrm{Re}_L^{1/5}} - \frac{1742}{\mathrm{Re}_L} \qquad Nu_L = \frac{\bar{h}L}{k} = \left(0.037\,\mathrm{Re}_L^{0.8} - 871\right)\mathrm{Pr}^{1/3}$$

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Figure 7-10 from Çengel, *Heat and Mass Transfer* 5

Review Cylinder and Sphere

- Cylinder average h (RePr > 0.2; properties at $(T_\infty + T_s)/2$

$$Nu = \frac{hD}{k} = 0.3 + \frac{0.62\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/2}}{\left[1+\left(\frac{0.4}{\mathrm{Pr}}\right)^{2/3}\right]^{1/4}}\left[1+\left(\frac{\mathrm{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

- Sphere average h (3.5 ≤ Re ≤ 80,000; 0.7 ≤ Pr ≤ 380; μ_s at T_s; other properties at T_∞)

$$Nu = \frac{hD}{k} = 2 + \left[0.4\,\mathrm{Re}^{1/2} + 0.06\,\mathrm{Re}^{2/3}\right]\mathrm{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$

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Review Tube Banks

Nusselt number correlations for cross flow over tube banks for $N > 16$ and $0.7 < Pr < 500$ (from Zukauskas, 1987)*

Arrangement	Range of Re_D	Correlation
In-line	0–100	$Nu_D = 0.9\, Re_D^{0.4} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	100–1000	$Nu_D = 0.52\, Re_D^{0.5} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	1000–2×10^5	$Nu_D = 0.27\, Re_D^{0.63} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	2×10^5–2×10^6	$Nu_D = 0.033\, Re_D^{0.8} Pr^{0.4} (Pr/Pr_s)^{0.25}$
Staggered	0–500	$Nu_D = 1.04\, Re_D^{0.4} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	500–1000	$Nu_D = 0.71\, Re_D^{0.5} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	1000–2×10^5	$Nu_D = 0.35 (S_T/S_L)^{0.2} Re_D^{0.6} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	2×10^5–2×10^6	$Nu_D = 0.031 (S_T/S_L)^{0.2} Re_D^{0.8} Pr^{0.36} (Pr/Pr_s)^{0.25}$

*All properties except Pr_s are to be evaluated at the arithmetic mean of the inlet and outlet temperatures of the fluid (Pr_s is to be evaluated at T_s).

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Table 7-2 from Çengel, *Heat and Mass Transfer*

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Review Internal Flow Basics

- The flow is confined
- There is a temperature and velocity profile in the flow
  - Use average velocity and temperature
- Wall fluid heat exchange will change the average fluid temperature
  - There is no longer a constant fluid temperature like T_∞ for computing heat transfer

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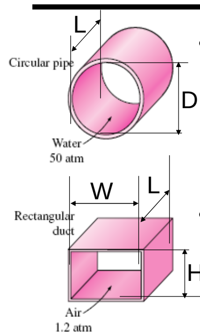
Review Area Terms



- A_{cs} is cross-sectional area for the flow
  - $A_{cs} = \pi D^2/4$ for circular pipe
  - $A_{cs} = LW$ for rectangular duct
- A_w is the wall area for heat transfer
  - $A_w = \pi DL$ for circular pipe
  - $A_w = 2(W + H)L$ for rectangular duct

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Figure 8-1 from Çengel, *Heat and Mass Transfer*

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Review Fixed Wall Heat Flux

- Fixed wall heat flux, $\dot{q}_{wall}$, over given wall area, A_w, gives total heat input which is related to $T_{out} - T_{in}$ by thermodynamics

$$\dot{Q} = \dot{q}_{wall} A_w = \dot{m} c_p (T_{out} - T_{in}) \quad \Rightarrow \quad T_{out} = T_{in} + \frac{\dot{q}_{wall} A_w}{\dot{m} c_p}$$

- "Outlet" can be any point along flow path where area from inlet is A_w
- We can compute T_w at this point as $T_w = T_{out} + \dot{q}_{wall}/h$

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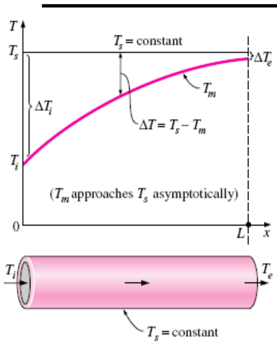
Review Constant Wall Temperature



$$(T_{out} - T_s) = (T_{in} - T_s) e^{-\frac{hA_w}{\dot{m} c_p}}$$

- $hA_w / \dot{m} c_p$ = NTU, the number of transfer units
- This is general equation for computing T_{out} in internal flows

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Figure 8-14 from Çengel, *Heat and Mass Transfer*

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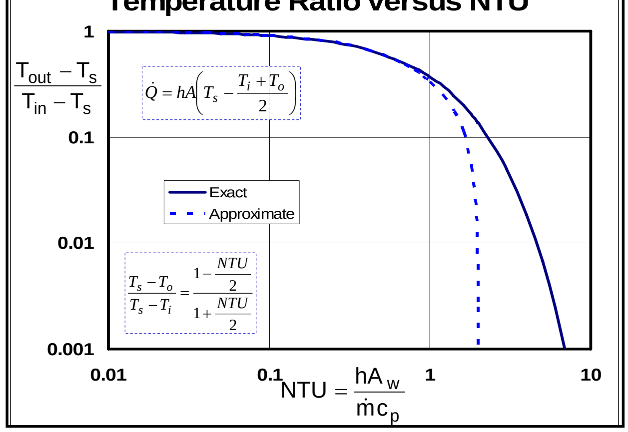
Temperature Ratio versus NTU



Review Log-mean Delta T

- Equations for overall heat transfer

$$\dot{Q} = \dot{m}c_p\left(T_{out} - T_{in}\right)$$

$$\dot{Q} = hA\left(LMDT\right)$$

$$LMDT = \frac{\left(T_{out} - T_{in}\right)}{\ln\left(\frac{T_{out} - T_s}{T_{in} - T_s}\right)} = \frac{\left(T_{out} - T_s\right) - \left(T_{in} - T_s\right)}{\ln\left(\frac{T_{out} - T_s}{T_{in} - T_s}\right)}$$

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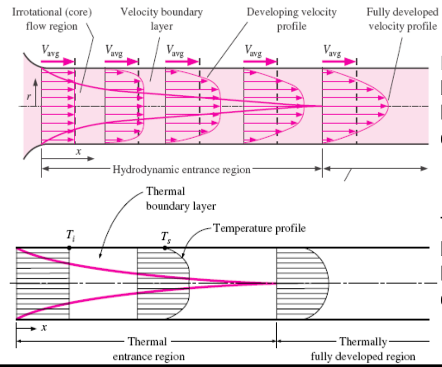
Developing Flows



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Fully Developed Flow

- Temperature profile does not change with x if flow is fully developed thermally
- This means that ∂T/∂r does not change with downstream distance, x, so heat flux (and Nu) do not depend on x
- Laminar entry lengths $\frac{L_h}{D} \approx 0.05\,\mathrm{Re}$ $\frac{L_t}{D} \approx 0.05\,\mathrm{Re\,Pr}$
- Turbulent entry lengths $\frac{L_t}{D} \approx \frac{L_h}{D} = 1.359\,\mathrm{Re}^{1/4} \approx 10$

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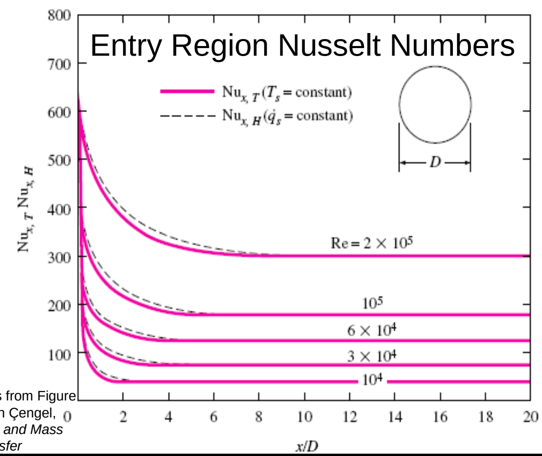
Entry Region Nusselt Numbers



Eggs from Figure 8-9 in Çengel, *Heat and Mass Transfer*

Internal Flow Pressure Drop

- General formula: Δp = f (L/D) ρV^2/2
- Friction factor, f, depends on Re = ρVD/μ and relative roughness, ε/D
- For laminar flows, f = 64/Re
  - No dependence on relative roughness
- For turbulent flows

Colebrook $\frac{1}{\sqrt{f}} = -2.0\log_{10}\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right)$

Haaland $\frac{1}{\sqrt{f}} \approx -1.8\log_{10}\left[\frac{6.9}{\mathrm{Re}} + \left(\frac{\varepsilon/D}{3.7}\right)^{1.11}\right]$

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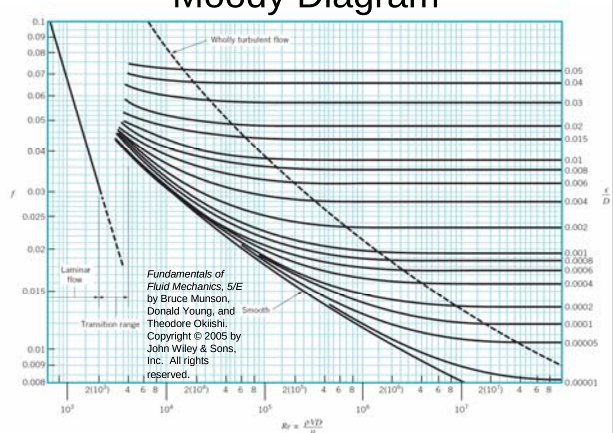
Moody Diagram



Fundamentals of Fluid Mechanics, 5/E by Bruce Munson, Donald Young, and Theodore Okiishi. Copyright © 2005 by John Wiley & Sons, Inc. All rights reserved.

Laminar Nusselt Number

- Laminar flow if Re = $\rho VD/\mu$ < 2,300
- Fully-developed, constant heat flux, Nu = 4.36
- Fully-developed, constant wall temperature: Nu = 3.66
- Entry region, constant wall temperature:

$$Nu = 3.66 + \frac{0.065(D/L)\mathrm{Re}\,\mathrm{Pr}}{1+0.04[(D/L)\mathrm{Re}\,\mathrm{Pr}]^{2/3}}$$

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Noncircular Ducts

- Define hydraulic diameter, $D_h = 4A/P$
  - A is cross-sectional area for flow
  - P is wetted perimeter
  - For a circular pipe where A = pD2/4 and P = πD, $D_h = 4(\pi D^2/4) / (\pi D) = D$
- For turbulent flows use Moody diagram with D replaced by D_h in Re, f, and ε/D
- For laminar flows, f = A/Re and Nu = B (all based on D_h) – A and B next slide

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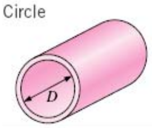
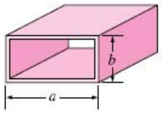
TABLE 8–1

Nusselt number and friction factor for fully developed laminar flow in tubes of various cross sections ($D_h = 4A_c/p$, $Re = V_{avg}D_h/\nu$, and $Nu = hD_h/k$)

Tube Geometry	a/b or θ°	Nusselt Number T_s = Const.	Nusselt Number $\dot{q}_s$ = Const.	Friction Factor f
Circle 	—	3.66	4.36	64.00/Re
Rectangle 	a/b			
	1	2.98	3.61	56.92/Re
	2	3.39	4.12	62.20/Re
	3	3.96	4.79	68.36/Re
	4	4.44	5.33	72.92/Re
	6	5.14	6.05	78.80/Re
	8	5.60	6.49	82.32/Re
	∞	7.54	8.24	96.00/Re

From Çengel, *Heat and Mass Transfer*

Turbulent Flow

- Smooth tubes (Gnielinski)

$$Nu = \frac{(f/8)(\mathrm{Re}-1000)\mathrm{Pr}}{1+12.7(f/8)^{0.5}(\mathrm{Pr}^{2/3}-1)} \quad \begin{pmatrix} 0.5 \le \mathrm{Pr} \le 2000 \\ 3x10^3 < \mathrm{Re} < 5x10^6 \end{pmatrix}$$

Petukhov : $f = [0.790\ln(\mathrm{Re}) - 1.64]^{-2} \qquad 3000 < \mathrm{Re} < 5x10^6$

- Tubes with roughness
  - Use correlations developed for this case
  - As approximation use Gnielinski equation with f from Moody diagram or f equation
    - Danger! h does not increase for f >$4f_{smooth}$

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Free (Natural) Convection

Forced

Free (Natural)

Eggs from Figure 1-33 in Çengel, *Heat and Mass Transfer*

- Flow is induced by temperature difference
  - No external source of fluid motion
  - Temperature differences cause density differences
  - Density differences induce flow
    - "Warm air rises"
- Used for electronic cooling with low cooling requirements

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Flow Direction

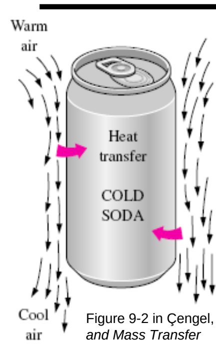

Figure 9-2 in Çengel, *Heat and Mass Transfer*

- Flow direction depends on temperature difference
  - Warm object (compared to ambient) causes fluid to rise
  - Cool object (compared to object) causes fluid to sink
  - Volume expansion coefficient: $\beta = -(1/\rho)(\partial\rho/\partial T)$
    - For ideal gases β = 1/T

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Laminar and Turbulent

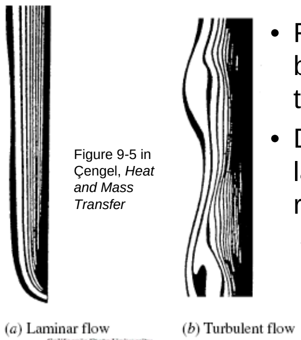

Figure 9-5 in Çengel, *Heat and Mass Transfer*

- Free convection can be laminar or turbulent
- Diagram shows laminar and turbulent regions
  - Mach-Zender interferometer shows density lines that are proportional to T

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Profiles

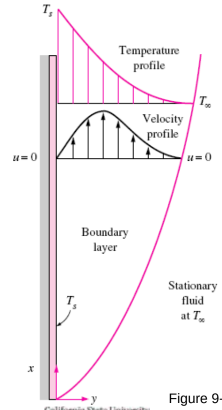

Figure 9-6 in Çengel, *Heat and Mass Transfer*

- Warm vertical wall at T_s with cooler fluid at T_∞
- Velocity is zero at wall and edge of boundary layer
- Driving force is density differences
  - $\rho = \rho_\infty + \beta(T - T_\infty)$

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Grashof and Rayleigh Numbers

- Dimensionless groups for free (natural) convection

$$Gr = \frac{\beta g \Delta T L_c^3}{\nu^2} = \frac{\rho^2 \beta g \Delta T L_c^3}{\mu^2} \qquad Ra = Gr\,\Pr = \frac{\beta g \Delta T L_c^3}{\nu\alpha}$$

  - g = acceleration of gravity (LT^{-2})
  - $\beta = -(1/\rho)(\partial\rho/\partial T)$ called the volume expansion coefficient (dimensions: $1/\Theta$)
  - $\Delta T = |T_{wall} - T_{fluid}|$ (dimensions: Θ)
  - Other terms same as previous use

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Equations for Nu

- Equations have form of $AGr^a Pr^b$ or BRa^c
- Since Gr and Ra contain $|T_{wall} - T_{fluid}|$, an iterative process is required if one of these temperatures is unknown
- Transition from laminar to turbulent occurs at given Ra values
  - For vertical plate transition Ra = 10^9
- Evaluate properties at "film" (average) temperature, $(T_{wall} + T_{fluid})/2$

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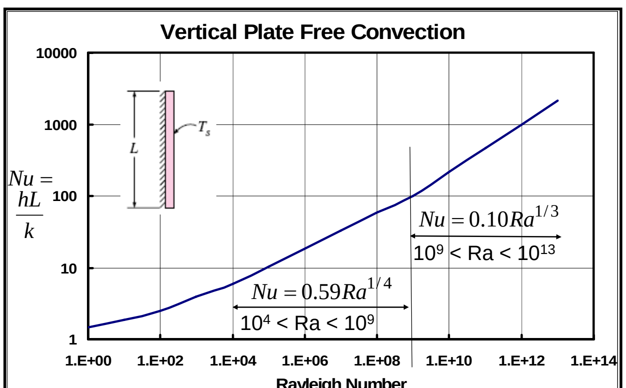

Plate figure from Table 9-1 in Çengel, *Heat and Mass Transfer*

$$Ra = \frac{\beta g \Delta T L^3}{\nu^2}\Pr = \frac{\beta g \Delta T L^3}{\nu\alpha}$$

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Vertical Plate Free Convection

- Simplified equations on previous chart for constant wall temperature
  - More accurate: Churchill and Chu, any Ra

$$Nu_L = \left\{0.825 + \frac{0.387 Ra_L^{1/6}}{\left[1 + (0.492/\Pr)^{9/16}\right]^{8/27}}\right\}^2 \qquad Any\ Ra_L$$

  - More accurate laminar Churchill/Chu

$$Nu_L = 0.68 + \frac{0.670 Ra_L^{1/4}}{\left[1 + (0.492/\Pr)^{9/16}\right]^{4/9}} \qquad 0 < Ra_L < 10^9$$

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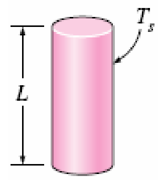
Vertical Plate Free Convection

- Constant wall heat flux
  - Use $\dot{q} = hA(T_w - T_\infty)$ to compute an unknown temperature (T_w or T_∞) from known wall heat flux and computed h
  - T_w varies along wall, but the average heat transfer uses midpoint temperature, $T_{L/2}$

$$\dot{q}_{wall} = hA_{wall}(T_{L/2} - T_\infty) \quad \Rightarrow \quad T_{L/2} - T_\infty = \frac{\dot{q}_{wall}}{hA_{wall}}$$

  - Use trial and error solution with $T_{L/2} - T_\infty$ as ΔT in Ra used to compute h = kNu/L

Vertical Cylinder



Cylinder figure from Table 9-1 in Çengel, *Heat and Mass Transfer*

- Apply equations for vertical plate from previous charts if $D/L \geq 35/Gr^{1/4}$
- For this D/L effects of curvature are not important
- Thin cylinder results of Cebeci and Minkowcyz and Sparrow available in ASME Transactions

Horizontal Plate

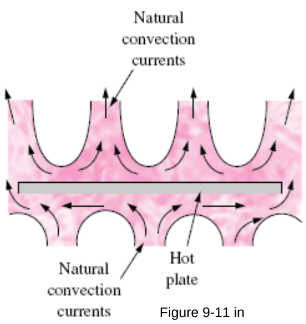


Figure 9-11 in Çengel, *Heat and Mass Transfer*

- Convection depends on temperature and direction
- Hot plate shown here has strong currents above plate and stops flow below plate

Horizontal Plate II

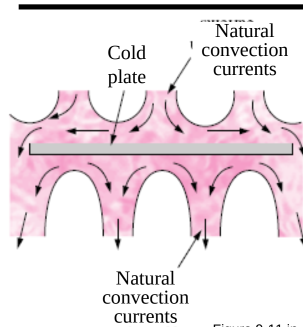
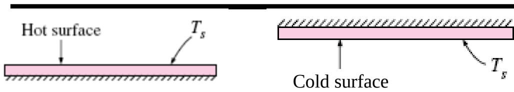


Figure 9-11 in Çengel, *Heat and Mass Transfer*

- Cold plate shown here ($T_{plate} < T_\infty$)
- This has strong currents below plate and stops flow above plate

Horizontal Plate III

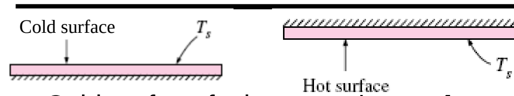


- Hot surface facing up or cold surface facing down
- L_c = area / perimeter (A_s/p)
  - For a rectangle of length, L, and width, W, $L_c = (LW) / (2L + 2W) = 1 / (2 / W + 2 / L)$
  - For a circle, $L_c = \pi R^2 / 2\pi R = R/2 = D/4$

Figures from Table 9-1 in Çengel, *Heat and Mass Transfer*

$$Nu = 0.54 Ra_{L_c}^{1/4} \quad 10^4 < Ra < 10^7$$
$$Nu = 0.15 Ra_{L_c}^{1/3} \quad 10^7 < Ra < 10^{11}$$

Horizontal Plate IV

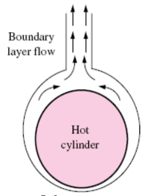


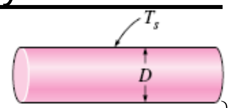
- Cold surface facing up or hot surface facing down
- L_c = area / perimeter (A_s/p)
  - For a rectangle of length, L, and width, W, $L_c = (LW) / (2L + 2W) = 1 / (2 / W + 2 / L)$
  - For a circle, $L_c = \pi R^2 / 2\pi R = R/2 = D/4$

Figures from Table 9-1 in Çengel, *Heat and Mass Transfer*

$$Nu = 0.27 Ra_{L_c}^{1/4} \quad 10^5 < Ra < 10^{11}$$

Horizontal Cylinder





$$Nu_D = \left\{0.6 + \frac{0.387 Ra_D^{1/6}}{\left[1+(0.559/\mathrm{Pr})^{9/16}\right]^{8/27}}\right\}^2$$

- Nu_D is average value for the cylinder
- Note differences around cylinder in figure on left
  - What if cylinder on left were cold?

Figure 9-12 (left) and figure from Table 9-1 (right) in Çengel, *Heat and Mass Transfer*

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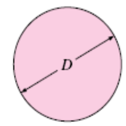
Sphere



- Equation is valid for $Ra_D <$ 10^{11} and Pr .7

$$Nu_D = 2 + \frac{0.589 Ra_D^{1/4}}{\left[1+(0.469/\mathrm{Pr})^{9/16}\right]^{4/9}}$$

Figure from Table 9-1 in Çengel, *Heat and Mass Transfer*

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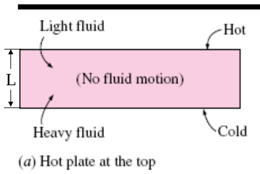
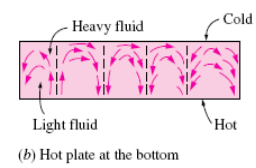
Horizontal Enclosures





- **Top side warmer:** no convection
- Conduction only, Nu = hL/k = 1
- **Bottom warmer:** convection becomes significant when $Ra_L = (\mathrm{Pr})\beta g \Delta T L^3/\nu^2 = \beta g \Delta T L^3/\nu\alpha > 1708$

Figure 9-22 in Çengel, *Heat and Mass Transfer*

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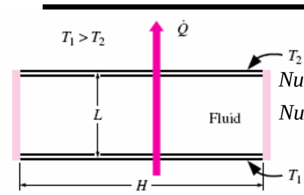
Horizontal Enclosures II



Jakob, for 0.5 < Pr < 2

$Nu = 0.195 Ra_L^{1/4}$ $10^4 < Ra_L < 4x10^5$

$Nu = 0.068 Ra_L^{1/3}$ $4x10^5 < Ra_L < 10^7$

Globe and Dropkin for a range of liquids

$Nu = 0.069 Ra_L^{1/3}\,\mathrm{Pr}^{0.074}$ $3x10^5 < Ra_L < 7x10^9$

Hollands *et al*. for air; also for other fluids if $Ra_L < 10_5$

$$Nu = 1 + 1.44\max\left(0, 1-\frac{1708}{Ra_L}\right) + \max\left(0, \frac{Ra_L}{18} - 1\right) \quad Ra_L < 10^8$$

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Vertical Enclosures

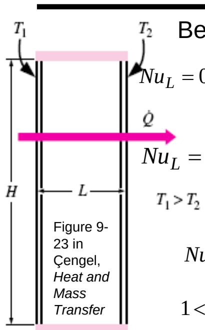

Figure 9-23 in Çengel, *Heat and Mass Transfer*

Berkovsky and Polevikov, any Pr

$Nu_L = 0.18\left(\frac{\mathrm{Pr}}{0.2+\mathrm{Pr}} Ra_L\right)^{0.29}$ $1 < H/L < 2$, $Ra_L \mathrm{Pr}/0.2 + \mathrm{Pr} > 10^3$

$Nu_L = 0.22\left(\frac{\mathrm{Pr}\, Ra_L}{0.2+\mathrm{Pr}}\right)^{0.28}\left(\frac{L}{H}\right)^{1/4}$ $2 < H/L < 10$, $Ra_L < 10^{10}$

MacGregor and Emery

$Nu_L = 0.42 Ra_L^{1/4}\,\mathrm{Pr}^{0.012}\left(\frac{L}{H}\right)^{0.3}$ $10 < H/L < 40$, $1 < \mathrm{Pr} < 2x10^4$, $10^4 < Ra_L < 10^7$

$Nu_L = 0.46 Ra_L^{1/3}$ $1 < H/L < 40$, $1 < \mathrm{Pr} < 20$, $10^6 < Ra_L < 10^9$

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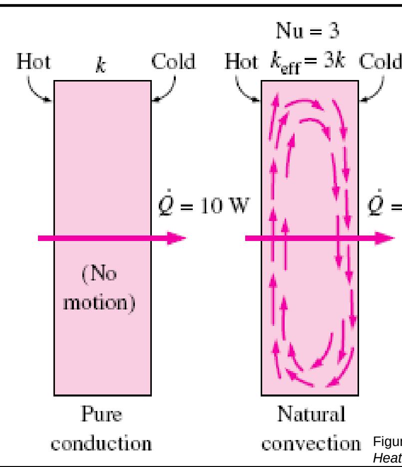
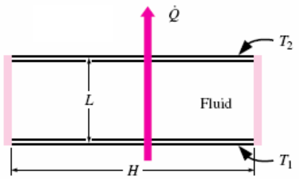
k_{eff}



- Effective thermal conductivity
- k_{eff} = kNu
- For vertical enclosures with no motion, k_{eff} = 1

Figure 9-23 in Çengel, *Heat and Mass Transfer*

Horizontal Enclosures



- If $T_2 > T_1$, (top warmer) there is no convection
- Conduction only, Nu = hL/k = 1

- If $T_2 < T_1$, (bottom warmer) significant convection starts when $Ra_L = (Pr)\beta gDTL^3/\nu^2 > 1708$

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